M. Sc. (Mathematics)

Semester I Paper IV (CC-04)

Discrete Mathematics

Boolean Algebra :→ A Boolean algebra of sets is a non-empty class $\mathcal{A}$ of subsets of the universal set U which has the following properties:

(i) $A, B \in \mathcal{A} \Rightarrow A \cup B \in \mathcal{A}$

(ii) $A, B \in \mathcal{A} \Rightarrow A \cap B \in \mathcal{A}$

(iii) $A \in \mathcal{A} \Rightarrow A' \in \mathcal{A}$.

$\mathcal{A}$ is assumed to be non-empty, so it contains at least one set A.

From, property (iii), A' is in $\mathcal{A}$ along with A.

$\because A \cap A' = \phi$ and $A \cup A' = U$

$\Rightarrow \mathcal{A}$ contains the empty set and the universal set.

Consider the collection C consisting only of the empty set and the universal set. Obviously it is a Boolean algebra of sets.

$\Rightarrow$ These two distinct sets are the only ones which every Boolea[n] algebra of sets must contain.

Another Example

The class (or collection) of all subsets of U is also a Boolean algebra of sets.

Let $\mathcal{A}$ be a Boolean algebra of sets.

Let $\{A_1, A_2, ---, A_n\}$ is a non-empty finite subclass of $\mathcal{A}$. Then

$A_1 \cup A_2 \cup --- \cup A_n$ and $A_1 \cap A_2 \cap --- \cap A_n$

are both sets in $\mathcal{A}$.

Since $\mathcal{A}$ contains the empty set and the universal set,

$\Rightarrow$ $\mathcal{A}$ is a class of sets which is closed under the formation of finite unions, finite intersections and complements.

Let $\mathcal{A}$ be a class of sets which is closed under the formation of finite unions, finite intersections and complements.

$\Rightarrow$ $\mathcal{A}$ ~~consists~~ contains the empty set and the universal set.

i.e. A is non-empty.

$\Rightarrow$ $\mathcal{A}$ is a Boolean algebra of sets.

Hence, Boolean algebras of sets can be described alternatively as classes of sets which are closed under the formation of finite unions, finite intersections and complements.